Measurement Based Self-Optimization in Random Access Communications

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Abstract

This work considers a single-cell random access channel in cellular wireless networks and provides an algorithmic approach to the problem of optimal coordination of user actions. In the scenario considered, an access effort is successful if (a) the signal is detected at the receiver and (b) no collision occurs. The first event is controlled by the user transmission power while the second one by the choice of access (back-off) probability. These constitute the user action pair. The algorithm aims at exploiting information from measurements and user reports, in order to estimate current values of the system situation. Based on these estimates, two optimization problems can be formulated and solved for the so called contention level and transmission power level at the base station side. The methodology to find the optimal values is based on minimization of a drift function. The two values are broadcast in order to help the users update their actions "almost optimally". In this way the wireless cell can achieve self-optimization, without external coordination, by relying on such intelligent information exchange and parameter estimation. Numerical results illustrate the great benefits of the proposed algorithm, compared to scenarios where the actions remain fixed, at a very low or even zero cost in power expenditure and delay.

Index Terms

Random Access Channel, Self Organizing Networks (SONs), Measurements, Collision Resolution, Drift Minimization, Power Control

I. INTRODUCTION

Random multiple access schemes have traditionally played an important role in wireless communication systems. Their use has been established especially in cases of bursty source traffic, where a multiplicity of users requires access to a central receiver. The most typical protocol is the celebrated ALOHA, first presented in the 1970's [1]. This has seen several modifications in the years to come, which aim especially at an increase in throughput performance. An interesting presentation of several related results can be found in [2]. Considering standardization, the IEEE 802.11 protocol for local area network communications has received a great deal of attention and extensive literature can be found related to its performance and optimization, see [3], [4], [5] and references therein. Random access standardization is also included in the 3rd Generation Partnership Project (3GPP) as an important element within the Long Term Evolution (LTE) systems [6].

The procedure is called *random access*, due to the fact that a number of sources aim at gaining access to a central entity, by randomly transmitting over the channel. In the general case, if more than one user transmit simulatneously, we say that a *collision* occurs and all efforts are considered unsuccessful. After a failure, each source enters a *back-off* mode for a time interval chosen based on some probability distribution. This is a time when the source remains silent in order to avoid a new collision. This back-off time can be modeled in the slotted case by a per slot probability of transmission, less than 1. Using this technique, an increase in throughput is achieved at the cost of additional delay.

Random access is by nature a decentralized scheme, where sources (users) should individually decide on an action, in this case to transmit or not. The decision making can be facilitated by the availability of certain global information over the system state, which could be broadcast by the central receiver. In this way the delay-throughput tradeoff is enhanced at the cost of additional signaling, due to message passing, as well as further computations and processing at both ends of the communication channel. Furthermore, such protocols allow users to dynamically adapt to changes in system state. Such an approach can be seen as a self-optimization technique for the involved system. Global information is gathered at the receiver end and after certain processing, cell-specific information is passed to the users who update their actions. The current paradigm change in cellular wireless systems towards self-organizing functionality in the so called SONs (Self Organizing Networks), with as few intervention as possible coming from system operators, encourages this approach.

Important role in a self-organizing algorithm for random access plays the type of information measured at the central entity, its processing, as well as the messages broadcast to the users. Such an idea can be already found for the IEEE 802.11 protocol in order to take service differentiation into account in [5]. A description of how random access works in the 3GPP-LTE systems together with suggestions

on the type of measurements and information extraction at the Base Station (BS) of an isolated cell is presented in [7]. On the other hand, a complete self-organizing algorithmic solution which is based on measurements and provides specific performance guarantees for a SON is, to the authors' knowledge, not available in the current literature. This work aims at covering this gap by developing a protocol with specific information exchange between users and BS, to guarantee a near-optimal performance related to a certain measure. The algorithm is based on the optimization of a so-called *drift* function, which allows the system to reach a *near optimal* steady state performance as $t \to \infty$, as will be explaned in the Performance Optimization Section to follow. The approach here is novel, borrows however certain ideas from the existing literature, which are shortly presented in what follows together with our contributions.

A. Related Work and Contributions

Bianchi [3] has been the first to provide a precise performance analysis for a random access protocol, which uses exponential back-off times. His approach considers a *saturated system model*, where the number of users is kept fixed to N and all have a packet to send at each time slot. The results are based on the key approximation that the collision probability of a packet transmitted is constant and independent, which decouples the evolution of the system to N 1-dimensional Markov Chains. This is because the contention probability is not considered here a function of the back-off probabilities. The approach performs well for a large number of users.

A different approach, which departs from the above approximation has been suggested by Sharma et al. [5], where more general back-off strategies (generalized geometric) are considered in a slotted random access protocol. One of the major differences is that the system state is described by the current number of users per effort, while the collision probability is not independent per user, so that the back-off probability assigned to a user affects the collision probability of the others as well. The system is again saturated and the throughput is provided by obtaining the unique equilibrium point of the drift equations associated with the back-off process. The analysis is valid for a large number of users, for which the associated Markov Chain stays close to a so called typical state.

In the above works, the back-off probabilities are not adapted dynamically. First suggestions for controlling multiple access protocols can be found in Hajek and van Loon [8]. Gupta et al [4] have recently suggested a dynamic back-off adaptation mechanism, where contention is regulated by broadcasting a so called contention level to the users, who update in turn their access probability based on this and their current effort index. The contention level is updated at the BS by a Multiplicative-Increase-Decrease (MID) rule, after each success or failure.

Cross-layer optimization using the drift minimization is presented in the tutorial of Dimic et al [9].

Other channel-aware scheduling approaches, in conjuction with random access mechanisms such as ALOHA can be found in [10], and more recently in [11] and references therein.

The random access procedure for the LTE system model is described by Amirijoo et al [7]. The major difference from the previous models is that the miss-detection probability of a transmission influences the system state evolution together with the contention probability. This can be controlled by the transmission power of the user. A power ramping function is suggested to differentiate between users based on their current effort index. Furthermore, the authors suggest the utilization of user reports at the BS, so that the contention and miss-detection probability are estimated at each time slot. Based on such estimates the BS can broadcast relevant information so that the users can update their actions.

In this work the problem formulation is based on a saturated system model, where the number of users remains constant within the cell. The LTE approach of the random access procedure is adopted, so that a transmission effort is successful when the signal is detected and no collision occurs. For the state evolution of the system we make *the assumption that the collision probability is the same for all users, but is a function of the entire vector of back-off probabilities chosen*. This assumption is in some sense related to the approach of Bianchi but does not decompose the system, it rather describes its evolution as an N-dimensional Markov Chain. The transition probabilities for each user are controlled by the action pair of access (or transmission) probability and chosen power. The model is presented in detail in Section II.

As performance measure, we consider in Section III the expectation of some function of the system state, as time goes to infinity. The function chosen for the analysis is the sum of the current effort indices of all users. It is shown that the optimal action pair per user can be found by solving a dynamic program. An approximation to the optimal solution is found by chosing actions myopically based on the drift minimization at each time slot.

The steps of the self-optimization algorithm are analytically presented in Section IV. The BS provides the users with two values, the current *contention level* and the current *power transmission level*, so that they can update their action pair. The first can be found as the solution of a reformulated drift minimization problem at the BS end. However certain unknown system quantities (the miss detection rate, contention rate as well as the number of users present within the cell) should be estimated and this is done based on measurements and user reports. The second level is updated based on a Multiplicative-Increase-Additive-Decrease (MIAD) rule.

Numerical results are shown in Section V. Our algorithm, which provides two possible choices for the contention level (high/low) and two actions (increase/decrease) for the power level, is compared to a fixed back-off and power level scenario, chosen such that a high performance is guaranteed for the

system. Our suggestion outperforms such a scheme at a certain cost of higher power expenditure (which appears however only when the number of users is relatively small) and additional delay. The costs are however trivial compared to the performance gains as illustrated in the simulation plots. Finally, Section VI concludes our work. Certain proofs of theorems presented in the main text can be found in the Appendix.

II. System Model

We consider an arbitrary but fixed total number of N users labeled by $n = \{1, \ldots, N\}$ trying to *randomly* obtain access to a cell base station over the wireless link. The time is slotted, with slot interval normalized to 1 and indexed by t. At each time slot all users belonging to the user set have the possibility to access the channel by transmitting e.g. a preamble sequence as in the LTE standards. We use the following notation for the events of interest (each corresponds to time slot t with the index omitted for brevity):

- PRN: a set of users with cardinality N is present within the cell
- TR(k\N): a user subset of cardinality k out of the total N transmits
- TRn: user n transmits
- D(l\N): a subset of cardinality l out of the total N transmitted user signals is detected
- Dn: user n's signal is detected
- COL: at least one pair of transmitting users collides
- COLn: user n takes part in a collision
- SUn: user n has a successful transmission
- DRn: user n is dropped after M consecutive failures
- IDN: all N users within the set do not transmit and the slot remains idle

The complements of the above events are denoted by a C index, e.g. Dn^C for the event of miss-detecting the n user signal at the BS.

There are two criteria that determine the success of an attempt.

A) The received signal at the BS should have a Signal-to-Noise Ratio (SNR) exceeding a predefined detection threshold γ_d. Otherwise, a miss-detection occurs and the user has to retry. The main reason for the miss-detection, is that the user's transmitted signal experiences random fading over the link. Another one could be a wrong choice of transmission power due to channel estimation errors, provided that such an estimate is available. The Detection Miss Probability (DMP) equals

$$\begin{aligned} DMP_n\left(p_n\left(t\right), \gamma_d\right) &:= \mathbb{P}\left[Dn^C | TRn\right] \\ &= \mathbb{P}\left[SNR_n\left(p_n\left(t\right), h_n\left(t\right)\right) \leq \gamma_d\right] \end{aligned} \tag{1}$$

where p_n is the chosen transmission power and the probability is taken over the random quantity, which can either be the actual channel fading coefficient or the fluctuation of the estimation error around a mean value. These two random variables are denoted both by h_n and they are independent identically distributed over the time slots t. We consider DMP functions with the following behavior

Property 1 *The detection miss probability $DMP_n\left(p_n\left(t\right),\gamma_d\right)$ is non-increasing with respect to the transmission power p_n and non-decreasing with respect to the threshold value γ_d.*

Notice that the property trivially holds in the widely-used case $SNR_n = \frac{p_n(t)\cdot h_n(t)}{\sigma^2}$, where h_n denotes the channel fading and σ^2 is the variance of zero-mean background noise. We point out that this case is of main interest in the analysis and algorithmic solutions to follow.

B) No *collision* of transmitted signals should occur. When more than one user attempts to access the channel during the same time slot, there is a probability that the BS cannot separate the two signals, in which case all affected users have to repeat the effort. The probability of collision is conditional on the detection of the signals at the BS side. The probability that at least one collision occurs, when N users transmit and l-out-of-N signals are detected is

$$CP\left(l,N,t\right) := \mathbb{P}\left[COL|D(l\setminus N)\ \&\ TR\left(N\setminus N\right)\right]. \tag{2}$$

In case no intelligent mechanism is available and the typical slotted ALOHA protocol is used [1], the probability of collision takes the value

$$CP\left(l,N,t\right) \overset{ALOHA}{=} \begin{cases} 0, & l\leq 1 \\ 1, & l>1 \end{cases}. \tag{3}$$

In other protocols, such as those suggested in LTE standards [12], a pool of orthogonal sequences (e.g. Zadoff-Chu) is made available to all users. Each user chooses one sequence from this set randomly (uniform distribution) and the probability of collision is less than 1. For example, when l-out-of-N users are detected and the sequence pool at time t contains $C\left(t\right)\geq l$ elements, the probability of a collision event can be found by solving the birthday problem [13]. We get then

$$CP\left(l,N,t\right) \overset{LTE}{=} \begin{cases} 0, & l\leq 1 \\ 1-\frac{C(t)!}{C^l(t)(C(t)-l)!}, & 1<l\leq C\left(t\right) \\ 1, & l>C\left(t\right) \end{cases}. \tag{4}$$

As a numerical example: If $l=2$ transmitting and detected users share a pool of $C\left(t\right)=5$ sequences, the collision probability equals 0.2.

The overall probability of a collision event when N users transmit, equals

$$CP\left(N,t\right) := \mathbb{P}\left[COL|TR\left(N\setminus N\right)\right] = \sum_{l=0}^{N} CP\left(l,N,t\right)\cdot\mathbb{P}\left[D\left(l\setminus N\right)\right]. \tag{5}$$

The following property is attributed to the collision probability expressions considered in this work. (It can be easily shown that the above examples satisfy it as well.)

Property 2 *The collision probability $CP\left(N,t\right)$ is non-decreasing in the number of transmitting users N, that is*

$$CP\left(N+1,t\right) - CP\left(N,t\right) \geq 0. \tag{6}$$

Remark 1 *In our model, the probability that a transmitting user n takes part in a collision, when N users transmit and his signal is detected, is set equal to the overall collision probability CP above*

$$CP_n\left(N,t\right) := \mathbb{P}\left[COLn|TR\left(N\setminus N\right)\ \&\ Dn\right] \overset{Assumption}{=} CP\left(N,t\right). \tag{7}$$

In other words, we assume that when at least one pair of all N transmitting users collides, all efforts are considered unsuccessful.

From the above, the probability of successful access to the BS, depends on the detection probability and the collision probability. Since the detection probability is influenced by the channel fading and the collision probability by the combined behavior of all the users, the success probability takes the following form

$$\begin{aligned} SP_n\left(p_n\left(t\right),\gamma_d,N,t\right) &\overset{(a)}{:=} 1-\mathbb{P}\left[SUn^C|\ TR(N\setminus N)\right] \\ &\overset{(b)}{=} 1-\mathbb{P}\left[Dn^C|\ TRn\right]-\mathbb{P}\left[Dn\ \&\ COLn|\ TR(N\setminus N)\right] \\ &\overset{Bayes}{=} \left(1-\mathbb{P}\left[Dn^C|\ TRn\right]\right)\cdot\left(1-\mathbb{P}\left[COLn|\ TR(N\setminus N)\ \&\ Dn\right]\right) \\ &\overset{(1),\ (5),\ (7)}{=} \left(1-DMP_n\left(p_n\left(t\right),\gamma_d\right)\right)\cdot\left(1-CP\left(N,t\right)\right). \end{aligned} \tag{8}$$

In the above, (a) comes from the definition of the success probability, which equals 1 minus the probability of failure given that N users transmit including n. The equality (b) comes from the fact that failure at a user effort occurs in the case of two events, either Dn^C, or Dn & COLn and the probability of their occurence is written as a sum. By applying Bayes' rule to the last term and reformulating, we reach the product form above. Taking into account the Remark 1 we reach the expression in (8).

There are *two actions* that user n can take for transmission at time slot t:

- The choice of the *transmission power level* $p_n(t)$, which influences the detection of the transmitted signal at the BS, as shown in (1).
- The choice of the *access (or transmission) probability* $b_n(t)$, which is the probability that user n transmits in a given time slot

$$b_n(t) \quad := \quad \mathbb{P}\left[TRn\right]. \tag{9}$$

This influences the number of simultaneously transmitting users in the cell and therefore directly affects the collision probability in (4). The *back-off* probability simply equals $1 - b_n(t)$.

The set of actions for the entire system of N users at t is denoted by the $2N$-dimensional vector $\mathbf{A}(t) := (\mathbf{b}(t), \mathbf{p}(t))$. The action space per time-slot is denoted by $\mathbb{A}$ and is the Cartesian product $[0,1]^N \times [0, P_1] \times \ldots \times [0, P_N]$, where P_n is a given individual user power constraint per slot. Furthermore, $\tilde{\mathbf{A}} = \{\mathbf{A}(1), \ldots, \mathbf{A}(t), \ldots\}$.

It is clear from Property 1 that, the higher the transmission power, the higher the success probability in (8). The influence of the back-off probability is, however, a bit more complex. First of all, observe that in the definitions (1), (5) and (8) all probabilities are conditioned on the event TRn. If no back-off action is taken, then $b_n(t) = 1$ and TRn is always true. On the other hand, a different choice of the access probability will have as a result that the actual success probability is given by the expression (8), multiplied by $b_n(t)$. Assigning $b_n(t) \leq 1$ to user n, displaces the users in time and the effect of collision is mitigated. Furthermore, since a set of users less or equal to N is simultaneously taking part at the access of the medium in some slot t, the collision probability will as well be reduced. This is explicitly stated in the following Proposition.

Proposition 1 *Given an access probability b_n per user, the collision probability $CP^{(b)}$ equals*

$$CP^{(b)}(N,t) \quad = \quad \sum_{k=0}^{N} CP(k,t) \cdot \left(\sum_{l=1}^{L(N,k)} \prod_{i=1}^{k} b_{q_l^{k.i}} \prod_{j=1}^{N-k} (1 - b_{\hat{q}_l^{k.j}}) \right) \tag{10}$$

where $CP(k,t)$ is defined in (5), q_l^k is a subset of k-out-of-N users with elements $q_l^{k.i}$, l is the index of the $L(N,k) = \begin{pmatrix} N \\ k \end{pmatrix}$ different k-combinations and $\hat{q}_l^k$ is the complementary user set with cardinality $N-k$. For the case where $b_n = b$, $\forall n$, the expression simplifies to

$$CP^{(b)}(N,t) \quad = \quad \sum_{k=0}^{N} CP(k,t) \cdot L(N,k) \cdot b^k (1-b)^{N-k}.$$

Furthermore, if $b_n \leq 1$, then we have

$$CP(N,t) \quad \geq \quad CP^{(b)}(N,t). \tag{11}$$

Each user is constrained to at most M *access efforts* until success and the efforts are indexed by m. When, after M efforts, a user has not succeeded in obtaining access, the user is considered discarded and replaced by a newcoming one, so that the total user number in the system always remains the same. The same holds when a user leaves the system after a successful transmission. Therefore, we say that the system is *saturated*. The number of users at transmission effort m in time slot t is denoted by $X_m(t)$ and from the above it follows that

$$\sum_{m=1}^{M} X_m(t) = N, \quad \forall t. \tag{12}$$

We occasionaly write in the following that a user at effort $m \in \{1, \ldots, M\}$ belongs to *user class* m.

III. Performance Optimization

A. System States and Transition Probabilities

We define the state of user n at slot t as the current transmission effort $S_n(t) \in \{1, \ldots, M\}$, whereas the system state as the N-dimensional vector

$$\mathbf{S}(t) = (S_1(t), \ldots, S_N(t)). \tag{13}$$

Altogether, there are M different user states and M^N different system states (e.g for a cell with 10 users and maximum 5 efforts, the number is approximately 10 million). The entire state space is denoted by $\mathcal{S}$. The system state forms an N-dimensional Markov chain, because whether or not a user transmits and whether or not the effort is successful does not depend on the past state history. The dependency among the state evolution of the individual users is related to the collision probability $CP^{(b)}$, which is a function of the entire access probability vector.

We group the transitions for each user into (a) returning to state 1 in case of transmission and success, (b) moving to the next effort in case of transmission and failure and (c) backing-off and remaining in the same state. The expressions for the transition probabilities are given below (only the time is used as argument in the probability related functions for ease of notation).

- For $1 \leq m < M$:

$$\mathbb{P}[S_n(t+1) = 1 | S_n(t)] = b_n(t)(1 - DMP_n(t)) \cdot \left(1 - CP^{(b)}(t)\right) \tag{14}$$

$$\mathbb{P}[S_n(t+1) = S_n(t) + 1 | S_n(t)] = b_n(t)\left(DMP_n(t) + CP^{(b)}(t) - DMP_n(t)\, CP^{(b)}(t)\right) \tag{15}$$

$$\mathbb{P}[S_n(t+1) = S_n(t) | S_n(t)] = 1 - b_n(t). \tag{16}$$

- For the user boundary state $m = M$:

$$\mathbb{P}\left[S_n\left(t+1\right)=1|S_n\left(t\right)=M\right] = b_n\left(t\right) \tag{17}$$

$$\mathbb{P}\left[S_n\left(t+1\right)=M|S_n\left(t\right)=M\right] = 1-b_n\left(t\right). \tag{18}$$

  A user in state M will either back-off, in which case he remains in the same state, or transmit. When a user transmits, he will either succeed or fail. In both cases the next state is set to 1, the user is removed from the system and is replaced by a new one so that the total number remains always equal to N. Observe that the transition probabilities in (17)-(18) for $m = M$ coincide with those for $m < M$, given by (14)-(16) when DMP_n and $CP_n^{(b)}$ is set to 0.

For the analysis it is further important to specify the *user dropping probability* which is given by

$$\begin{aligned}\mathbb{P}\left[DRn\right] &= \mathbb{P}\left[DRn|S_n\left(t\right)=M\right]\cdot\mathbb{P}\left[S_n\left(t\right)=M\right]\\ &= b_n\left(t\right)\cdot\left(DMP_n\left(t\right)+CP^{(b)}\left(t\right)-DMP_n\left(t\right)\cdot CP^{(b)}\left(t\right)\right)\cdot\mathbb{P}\left[S_n\left(t\right)=M\right]\end{aligned} \tag{19}$$

To calculate the steady state probabilities, the $M^N \times M^N$ transition probability matrix should be formed. Since the number of states is finite, and for each user the probabilities using the expressions (14)-(16) and (17)-(18) sum up to $\sum_{m=1}^{M}\mathbb{P}\left[S_n\left(t+1\right)=m|S_n\left(t\right)\right]=1$ (stochastic matrix), a steady state with probability sum equal to 1 always exists, although certain states may be transient and have zero probability [14].

B. Methodology and Approximation

Let $V\left(\mathbf{S}\left(t\right)\right)$ be a non-negative function of the system state and let $\mathcal{M}\left(V,\tilde{\mathbf{A}}\right)$ be a performance measure related to the steady state reached when $t\rightarrow\infty$

$$\mathcal{M}\left(V,\tilde{\mathbf{A}}\right) := \lim_{t\rightarrow\infty}\mathbb{E}\left[V\left(\mathbf{S}\left(t\right)\right)|\mathbf{S}\left(0\right)\right]. \tag{20}$$

We will use the notion of *drift* to describe the expected change of the function V from time slot t to $t+1$, given that the state of the system at t is $\mathbf{S}\left(t\right)$ and the action set taken is $\mathbf{A}\left(t\right)$. The latter defines the system state transition probabilities $p_{s_t\rightarrow s_{t+1}}$ and is therefore an argument of the drift function

$$D\left(V\left(\mathbf{S}\left(t\right)\right),\mathbf{A}\left(t\right)\right) := \mathbb{E}\left[V\left(\mathbf{S}\left(t+1\right)\right)-V\left(\mathbf{S}\left(t\right)\right)|\mathbf{S}\left(t\right)\right]. \tag{21}$$

The significance of the drift will be clear from the following propositions.

Proposition 2 *The performance measure can be written as an infinite sum of expected drifts over the discrete time axis, given the initial state* $\mathbf{S}(0)$

$$\mathcal{M}\left(V, \tilde{\mathbf{A}}\right) = V(\mathbf{S}(0)) + \sum_{t=0}^{\infty} \mathbb{E}\left[D\left(V\left(\mathbf{S}(t)\right), \mathbf{A}(t)\right) | \mathbf{S}(0)\right]. \tag{22}$$

Proposition 3 *The problem of minimizing the system's performance measure, by choosing at each time slot* t *an appropriate action*

$$\begin{aligned} \textbf{\textit{min}} \quad & \mathcal{M}\left(V, \tilde{\mathbf{A}}\right) \\ \textbf{\textit{s.t.}} \quad & \mathbf{A}(t) \in \mathbb{A},\ t = 0, 1, \ldots \end{aligned} \tag{23}$$

can be solved as a dynamic program. The optimal solution satisfies Bellman's equation [15]

$$J(\mathbf{S}) \;=\; \min_{\mathbf{A} \in \mathbb{A}} \left\{ D\left(V(\mathbf{S}), \mathbf{A}\right) + \sum_{\mathbf{S}' \in \mathcal{S}} p_{s \to s'} J(\mathbf{S}') \right\},\ \forall \mathbf{S} \in \mathcal{S} \tag{24}$$

for the cost-to-go function $J(\mathbf{S})$*, where* $\mathbf{S}'$ *is the possible state at the next time slot, while the transition probabilities* $p_{s \to s'}$ *are functions of the actions chosen. The solution is state-dependent, meaning that the optimal actions depend on the system state and not on time.*

Proposition 4 *The solution of the drift minimization problem at each time slot* t

$$\begin{aligned} \textbf{\textit{min}} \quad & D\left(V\left(\mathbf{S}(t)\right), \mathbf{A}(t)\right) \\ \textbf{\textit{s.t.}} \quad & \mathbf{A}(t) \in \mathbb{A} \end{aligned} \tag{25}$$

is a suboptimal solution to the original problem in (23). The solution is called one-stage look-ahead (myopic), in the sense that the actions are chosen at each slot, considering only the transition to the next state and not the entire cost-to-go.

In the following paragraphs, we approach the performance measure optimization problem by minimizing the drift per time slot, described above. Although it is a suboptimal solution for the original problem, it can however lead to tremendous performance gains, compared to non-optimized schemes where the actions are given a fixed value valid for all time slots. It can furthermore allow for an implementation based on measurements and the ideas of self-optimization.

C. Performance Measure under Study and State-Dependent Actions

In the current work, the function V to be used is the sum of user states, which can be rewritten as the sum of cardinalities of users at a state, weighted by their effort index.

$$V(\mathbf{S}(t)) = \sum_{n=1}^{N} S_n(t) = \sum_{m=1}^{M} m \cdot X_m(t) \tag{26}$$

In this sense, a user who is currently at a higher effort, adds more to the function, than users in lower ones. Minimizing such a measure, reduces both the expected number of users at higher efforts and the drop probability, which by (19), depends on the steady state probability of a user being at state M.

We consider a homogeneous Markov chain by choosing the actions *state-dependent* rather than time-dependent. The motivation comes from the equation in (24), where the actions are functions of the state only. We write $\mathbf{A}(\mathbf{S}(t)) = (\mathbf{b}(\mathbf{S}(t)), \mathbf{p}(\mathbf{S}(t)))$. Under such an approach the probabilities affecting the state transitions can be grouped into classes indexed by m, rather than provided individualy for each user with index n

$$\begin{aligned} DMP_n(p_n(t), \gamma_d) \rightarrow &\quad DMP_n(p_n(S_n(t)), \gamma_d) \rightarrow \quad DMP_m(t) \\ b_n(t) \rightarrow &\quad b(S_n(t)) \rightarrow \quad b_m(t) \\ CP^{(b)}(N,t) \rightarrow &\quad CP^{(b)}(\mathbf{S}(t), N, t) \rightarrow \quad CP^{(b)}(N,t). \end{aligned}$$

The collision probability remains $CP^{(b)}(N,t)$, since it has by Remark 1 the same value for all classes and is time-dependent according to the way the N users are distributed among the M classes. With these definitions and observations, the drift expression can now be derived to yield

$$\begin{aligned} D(V(\mathbf{S}(t)), \mathbf{A}(t)) &= \sum_{n=1}^{N} \{1 \cdot \mathbb{P}[S_n(t+1) = 1 | S_n(t)] + \\ &\quad (S_n(t)+1) \cdot \mathbb{P}[S_n(t+1) = S_n(t)+1 | S_n(t)] + \\ &\quad S_n(t) \cdot \mathbb{P}[S_n(t+1) = S_n(t) | S_n(t)] - S_n(t)\} \\ &\overset{(14)-(16)}{=} \sum_{n=1}^{N} b_n(t) \cdot \left[1 - S_n(t) \cdot (1 - DMP_n(p_n(t), \gamma_d)) \cdot \left(1 - CP^{(b)}(N,t)\right)\right] \\ &\overset{state\_dep.}{=} \sum_{m=1}^{M} X_m(t) \cdot b_m(t) \cdot \left[1 - m \cdot (1 - DMP_m(t)) \cdot \left(1 - CP^{(b)}(N,t)\right)\right] \end{aligned} \quad (27)$$

IV. Measurement Based Self-Optimization

In order for the system of users and base station to have near-optimal performance with respect to $\mathcal{M}\left(V, \tilde{\mathbf{A}}\right)$, problem (25) should be solved per time slot at the base station side, with the drift given by (27) and the optimization variables being the state-dependent actions $\mathbf{A}(\mathbf{S}(t))$. These actions should then be transmitted as information to the user set. For the base station to solve the problem, it follows from (27) that it may have to access to the following information:

1) The cardinality $X_m(t)$ of users at effort m in slot t, as well as the sum of present users in the cell $\sum_m X_m(t) = N(t)$.

2) The expression for DMP_m at effort m as a function of transmission power $p_m(t)$, or the actual value of this probability.
3) The expression for $CP^{(b)}$ as a function of N and $\mathbf{b}(t)$, or the actual value of this probability.

The concept of *self-optimization* in wireless networks means that the base station adapts its functionality in an optimal way to changes that are out of its direct control. In the case of random access channels, such changes may include a drastic increase or decrease in the number of users present in the cell, varying fading conditions or bursts which can not be modeled easily by a priori assumptions on statistical distributions and their moments, unpredictable user mobility etc. For all these reasons, the number of users, DMP and CP are in general unknown variables.

The base station, however, may estimate the variables and with it approximate the objective function which results from a specific analytical model presented in the previous paragraphs, using *measurements* on channels and service quality, as well as *information, reported directly by the user set*. The goal is to use these estimates for optimization, in order to achieve significant performance gains, while keeping an additional overhead as small as possible. The general five steps of the proposed self-optimization algorithm in this work are summarized in Table I.

The subsections to follow, explain each step in detail. Before proceeding, we make certain assumptions, considering the action pair of access probabilities and transmission powers, so that the solution of the resulting problems is simplified and the required message passing between users and the BS is kept relatively low:

- The access probabilities are, as already mentioned, actions to be chosen optimally to minimize the drift at each time slot. The result is broadcast over the channel in the future steps of the algorithm, so that the users can adapt their transmissions in an optimal way. The approach in [4] is adopted here, where the per effort probability is set to
$$b_m(t) = \min\left\{\frac{f(m)}{L(t)}, 1\right\}, \ \forall m. \tag{28}$$
Here and hereafter, L is called *contention level* and $f(m)$ is some fixed function of the transmission effort. In this way, a simple variable L can simultaneously define the entire set of transmission probabilities. By choosing f to be monotone increasing in m, priority is given to users with higher efforts, while such users obtain lower priorities when f is strictly monotone decreasing. Typical back-off protocols reduce by half the probability of accessing the channel after each failure, so in this case $f(m) = 2^{-m+1}$ and $b_1 = 1/L$. Other possible choices could be $f(m) = m^{-1}$ (which is mostly used in this work and the simulations to follow) or $f(m) = m^{-0.5}$. In the following, the expression in (28) will sometimes be replaced by $b_m(t) = f(m)/L(t)$ and the constraint $b_m(t) \leq 1$ is taken into account in the constraint set of the minimization problem.

- We consider, furthermore, the transmission power to vary per effort as a ramping function. This approach is often considered in practice (for related approaches, the reader is referred to [7] and references therein). The power level for the first effort is given by P and for all efforts by the expression

$$p_m(t) \quad = \quad P(t) + (m-1) \cdot \Delta p, \ \forall m \tag{29}$$

  where Δp is the ramping step with a fixed (tunable) value. Thus, analogously as in the case of the backoff probabilities, the vector of power actions can be defined by appropriate choice of the *power level* $P(t)$ per time slot.

A. Step 1: Measurements and User Reports

When users attempt to randomly access the channel, we assume that the BS counts the overall number of detected user efforts, as well as the overall number of successful efforts. Given an observation window of length W, both the quantities depend on the time interval $[t-W+1, t]$ and are denoted by $N_d(t)$ and $N_s(t)$ respectively. Furthermore, after every successful effort, the users are assumed to *report* to the BS, the total number of trials required to get access. In this way, the BS can keep track of the number of successes at effort m, within the observation window, denoted by $n_{s,m}(t)\,,\ \forall m$. The reports over the success state also provide information over the overall number of transmissions of users being at some state m. As an example, if within the observation period two users report success at effort 3 and 2 respectively, the BS can estimate the number of transmissions at state $m=1$ by 2, at $m=2$ by 2 and at state $m=3$ by 1, without considering users that have yet not declared success, or are dropped. We denote these estimates by $n_{t,m}(t)\,,\ \forall m$ and their sum, which equals approximately the number of access efforts within the observation window, by $N_t(t) = \sum_{m=1}^{M} n_{t,m}$. Altogether, the set of gathered empirical information, updated per time slot, is represented by

$$\mathcal{I}(t) := \{N_d(t),\ N_s(t),\ N_t(t),\ n_{s,m}(t), \forall m,\ n_{t,m}(t), \forall m\} \tag{30}$$

B. Step 2: Estimation of Unknowns in the Objective function

Using the above counters, we can now approximate the unknowns in the expression (27) that are briefly discussed in points 1)-3) at the beginning of this section.

- As far as the unknowns in 2) and 3) are concerned, the actual overall contention probability $CP^{(b)}(N,t)$ defined in (5), (10) and per effort success probability $SP_m(t)$ in (8), can be estimated

by contention and success rates, an idea which has already appeared in [7]. Observe that the additional information about the per effort DMP_m cannot be deduced from the above measurements. What can be calculated, instead, is an overall rate of miss-detection, without differentiating between efforts, which we denote by DMR. Then,

$$\begin{aligned} CR(t) &= 1-\frac{N_s(t)}{N_d(t)} && (31)\\ SR_m(t) &= \frac{n_{s,m}(t)}{n_{t,m}(t)}, \forall m && (32)\\ DMR(t) &= 1-\frac{N_d(t)}{N_t(t)}. && (33)\end{aligned}$$

- Regarding the number of users currently within the cell - discussed in 1) - and their estimation, we proceed as follows. Instead of attempting to find integer values, we consider arrival rates. As arrival rates of users, we understand the number of users within the cell, divided by the length of the observation window. In case their number was a priori known somehow, this would equal the ratio $\frac{N}{W}$ for the whole system and $\frac{X_m}{W}$ for each one of the classes. Since we aim at minimizing the drift by an appropriate choice of actions, dividing (27) by W has no impact on the optimal solution.

  As rate of user arrivals, we consider the ratio $\frac{N_s(t)}{W}$, which is the time dependent ratio of accepted users, divided by the observation window. The above is used under the assumption that only a very small fraction of the users are dropped throughout the process, so that almost all users appearing within the cell, will eventually have at some point a success. Taking dropped users into account requires an additive correcting term that may be deduced from empirical observations.

  The window is considered long enough, so that the resulting success rates per state, $SR_m(t)$ in (32), approach the actual success probability per effort. These can replace the entries in the one-step transition probability matrix in equations (14)-(16) and (17)-(18). For example in the case $M=5$, we obtain (ommiting time dependency for brevity and assuming that the values remain almost fixed for a certain long enough time interval)

$$\hat{\mathbf{P}}_5 := \begin{bmatrix} 1-b_1(1-SR_1) & b_1(1-SR_1) & 0 & 0 & 0\\ b_2SR_2 & 1-b_2 & b_2(1-SR_2) & 0 & 0\\ b_3SR_3 & 0 & 1-b_3 & b_3(1-SR_3) & 0\\ b_4SR_4 & 0 & 0 & 1-b_4 & b_4(1-SR_4)\\ b_5 & 0 & 0 & 0 & 1-b_5 \end{bmatrix}.$$

Note that the steady state probability distribution is found by solving the system $\pi = \pi \cdot \hat{\mathbf{P}}_{\mathbf{M}}$, where π is the row vector of the unknown probabilities for the M states. It may be verified that the solution is equal to

$$\pi_1(t) = \frac{1}{1 + \sum_{i=2}^{M} \frac{b_1}{b_i}(1 - SR_1(t)) \cdot \ldots \cdot (1 - SR_{i-1}(t))} \tag{34}$$

$$\pi_m(t) = \pi_1(t) \cdot \left(\frac{b_1}{b_m}(1 - SR_1(t)) \cdot \ldots \cdot (1 - SR_{m-1}(t)) \right), \ 2 \leq m \leq M. \tag{35}$$

The ratios of the unknown backoff probabilities b_1/b_m are involved in the expression above. From the previous discussion $b_1/b_m = f(1)/f(m)$, which is known since the function f is chosen a priori. With these observations and definitions at hand, we can estimate the user arrivals per effort according to

$$\frac{X_m(t)}{W} \approx \pi_m(t) \cdot \frac{N_s(t)}{W} \tag{36}$$

where the π_m's are the probabilities given by (34) and (35).

C. Step 3: Solving the Problem

Once step 2 is performed, we can formulate the objective function to approximately solve problem (25) and with it find the optimal actions per time slot. To this end, we break down the problem into two subproblems and propose two sub-algorithms based on the measurements and estimated quantities described above.

- **Backoff Probability Problem**: The objective function at the base station is estimated by

$$\hat{D}(V(\mathbf{S}(t)), L(t)) := \frac{1}{L(t)} \cdot \left[\sum_{m=1}^{M} \pi_m \frac{N_s(t)}{W} f(m) \cdot (1 - m \cdot SR_m(t)) \right]. \tag{37}$$

The goal is to minimize the above convex function over the contention level L. Obviously, the optimization will have as a result either maximum or minimum value of L depending on the sign of the term inside the square brackets. The lower bound on L follows from the fact that all access probabilities are less than or equal to 1:

$$\frac{f(m)}{L(t)} \leq 1, \ \forall m \quad \Rightarrow \quad L(t) \geq \max\{f(m)\}.$$

To obtain an upper bound, we further provide a constraint on the probability of a time slot being idle (no user transmits). This probability is less than or equal to $\mathcal{A}$, which is a design factor for the system. Hence, we have

$$\mathbb{P}\left[IDN\right] = \prod_{m=1}^{M}\left(1-\frac{f(m)}{L\left(t\right)}\right)^{\frac{X_m(t)}{W}} \leq \mathcal{A} \;\;\Rightarrow\;\; \sum_{m=1}^{M}\pi_m\frac{N_s\left(t\right)}{W}\cdot\log\left(1-\frac{f(m)}{L\left(t\right)}\right) \leq \log(\mathcal{A}).$$

The function on the left handside is increasing with L, thus the inequality leads to an upper bound on L. The necessary and sufficient conditions for optimality of the contention level are summarized as follows

Proposition 5 *Considering the problem of minimizing $\hat{D}$ in (37) subject to the upper and lower bound constraints on L presented above, the following necessary and sufficient optimality conditions hold:*

- *if $\left[\sum_{m=1}^{M}\pi_m\frac{N_s(t)}{W}f\left(m\right)\cdot\left(1-m\cdot SR_m\left(t\right)\right)\right] \geq 0$ then the contention level is found by solving*

$$\sum_{m=1}^{M}\pi_m\frac{N_s\left(t\right)}{W}\cdot\log\left(1-\frac{f(m)}{L^*\left(t\right)}\right) \;=\; \log(\mathcal{A}) \tag{38}$$

- *if $\left[\sum_{m=1}^{M}\pi_m\frac{N_s(t)}{W}f\left(m\right)\cdot\left(1-m\cdot SR_m\left(t\right)\right)\right] < 0$ then the contention level equals*

$$L^*\left(t\right) = \max\left\{f(m)\right\}. \tag{39}$$

- **Power Control Problem**: In order to identify optimal transmission levels, one could proceed along similar lines as above, to formulate an optimization problem, given the backoff probabilities $f(m)/L^*(t)$ and the contention rates $CR(t)$ from (31). In order to determine the objective function based on (27), which is denoted by $\tilde{D}\left(V\left(\mathbf{S}\left(t\right)\right),P\left(t\right)\right)$, the closed form expression for the DMP_n as a function of power, with Property 1, may be necessary. It is however unlikely that the channel's fading behavior in practical systems can be accurately represented by a closed-form expression.
  A different approach - which is adopted here - is to use a *Multiplicative-Increase-Additive-Decrease* (MIAD) control rule, as in the case of congestion control protocols in TCP [16]. In this way, the BS reacts to the change of the estimated DMR stepwise, by increasing or decreasing the power level $P(t)$ per time slot, depending on the current value $DMR\left(t\right)$. We set two levels of action, a high detection-miss level DMR^H and a low one DMR^L. The control loop then works as follows: When the high level is exceeded, the power level is increased by multiplication with a tunable factor $1+\delta_1$. This action increases considerably the transmission power since miss-detection is highly non-desirable. When the ratio falls under the low level DMR^L, which is considered satisfactory

for the system performance, the power is reduced in a conservative way, to reduce the energy consumption on the mobile devices, by subtracting a constant tunable amount of δ_2. For instance δ_2 can be set equal to the ramping step Δp in (29). The control loop is then described by the following power updates

$$P^*(t) = \begin{cases} P^*(t-1)\cdot(1+\delta_1), & \text{if } DMR(t) > DMR^H \\ P^*(t-1) - \delta_2, & \text{if } DMR(t) < DMR^L \end{cases}. \tag{40}$$

D. Step 4 and 5: Broadcast of Information to the Users and Action Calculation

The last two steps of the proposed algorithm involve the broadcasting of the action-related information to the users and the choice of appropriate actions by them. The broadcast information includes the pair consisting of the contention level and the power level

$$\mathcal{J}(t) := \{L^*(t), P^*(t)\}. \tag{41}$$

Let us assume that the expressions in (28) and (29) for the success probability and the power level per effort are known a priori to the mobile stations. Since each user is aware of his current individual state $S_n(t)$, he can calculate his own action pair accoarding to

$$A_n(S_n(t), \mathcal{J}(t)) = (b_n(t), p_n(t)) = \left(\frac{f(S_n(t))}{L^*(t)},\ P^*(t) + S_n(t)\Delta p\right). \tag{42}$$

Note that if the required information is not available at the mobiles, the BS could broadcast the entire vector of computed transmission powers and access probabilities to the mobile users so that they choose the actions according to their current state.

The block diagram of the suggested algorithmic procedure is illustrated in Fig.1.

V. Numerical results

The proposed algorithm has been implemented in a single cell scenario. The number of users within the cell at each time slot is considered fixed throughout each implementation period and users are randomly positioned, with a 2D uniform distribution. The algorithm is evaluated for the cases of $N = 1, 2, \ldots, 14$ [users/time slot].

Considering the transmission scenario, each user randomly chooses at each attempt one sequence, out of a pool of 10 orthogonal sequences (we implement a simpler version of the LTE system model for collision avoidance), and transmits with a chosen backoff probability and transmission power. The

signal experiences path loss due to the user-BS distance. Fast fading is not modeled here but the channel is considered AWGN with noise mean equal to -133.2 dB.

An effort is successful when among the detected sequences there exists no pair that collides, in the sense that no two detected users choose the same sequence for transmission. A user is dropped when the effort fails at the maximum access effort $M = 5$. After a success or an event of dropping, users will be removed from the waiting-for-transmission list, and the same number of newly arriving users will be added, each given a random position on the plane.

Power and access probability for the users are computed per slot equal to the action pair in (42), for $f(m) = m^{-1}$. The suggested algorithm is compared to a scenario, where access probabilities and target power for each effort are held fixed, while the ramping function for the transmission power is the same in both cases. To show the effect of the design factor $\mathcal{A}$, two different values are tested in the suggested algorithm. The parameters of the two schemes are summarized in Table II and the performance comparisons are illustrated in Fig. 2 - 10.

The performance of the scheme and its comparison to the fixed scenario is better shown in the plots of the performance measure in Fig.2 and the dropped user rate in Fig.3. The performance measure is simply the actual value of the state function V in (26), for large numbers of time t, when the system is considered to have converged to a steady state. To make the comparison more "fair", the fixed scenario is chosen with access probability vector, such that the average occurence of an idle slot is equal to the case where $\mathcal{A} = 0.25$. This is higher for the scenario where $\mathcal{A} = 0.5$ as shown in Fig.4. For both choices of $\mathcal{A}$ the suggested algorithm outperforms the fixed scenario. The value of the performance function is lower (better)- with increasing difference to the fixed case, for a higher number of users. Considering the dropping user rate, the dynamic scenario allows the system to remain stable - in the sense that the rate of dropped users does not tend to "explode" - for a higher value of N.

A more detailed comparison of the schemes is given in Fig.5-10. Specifically, Fig.5, 6 and 7 illustrate three key performance indicators: the detection-miss, contention and success rate respectively. Observe how the increase of the parameter $\mathcal{A}$ improves all these, as well as the average effort for success, shown in Fig.8.

The last two figures provide the cost comparison for the two schemes, related to the average total number of time slots until success, as well as the expense in power for miss-detection avoidance. Higher benefits in the contention and success rate, naturally come at the cost of higher average delay up to success, as Fig.9 implies. On the other hand, the cost in average power expenditure per user equipment in Fig.10 is not necessarily higher in the dynamic scheme resulting by the algorithm, and can actually be less than in the fixed case, especially for a larger number of users in the cell.

Altogether, the dynamic scenario shows a significant increase in performance for the system at low costs in power and delay (sometimes even better), compared to a reasonable fixed scenario which aims at high performance. This is because the algorithm allows the system to react to performance degradation, observed by the measured DMR and CR, by choosing the two different actions for the power as well as the access probabilities adaptively. Obviously, availability of a larger action set would lead to even larger improvement, at the cost however of significant signaling overhead from the BS to the users.

VI. Conclusions

We have suggested an algorithm for decentralized control of user back-off probabilities and transmission powers in random access communications. The algorithm is based on measurements and user reports at the base station side, which allow for an estimation of the number of users present within the cell, as well as the quanitites of detection-miss and contention probability. By solving a drift minimization problem for the contention level and using closed loop updates for the transmission power level by a MIAD rule, the BS coordinates the actions chosen by the users, by broadcasting the information pair $(L^*(t), P^*(t))$.

The algorithmic steps, together with the methodology of the drift minimization for a certain measure of interest referring to the steady state, provide a general suggestion to treat problems of self-organization in wireless networks.

Considering the specific scheme, a large variation of algorithms can be extracted, by choosing e.g. some different state function for the performance measure, or by introducing other kinds of user reports, which may provide more information to the central receiver, at the cost of increase in signaling. Furthermore, a larger action set can definitely provide a higher performance, compared to the proposed one - which introduces two possible values for the contention level (high/low) and two actions for the power level (increase/decrease). Even in this scheme however, which is characterized by an "economy" of signaling and information exchange, the results - as illustrated by numerical examples - are extremely beneficial, especially as the user number in the cell increases.

Appendix

Proof: **[Proposition 1]** Similarly to the definition of CP in (5), the collision probability with N users present within the cell, each one of which choses to transmit or not based on a probability b_n, takes the form

$$\begin{aligned} CP^{(b)}(N,t) &:= \mathbb{P}[COL|\ PRN] \\ &= \sum_{k=0}^{N} CP(k,t) \cdot \mathbb{P}[TR(k \setminus N)] \end{aligned} \tag{43}$$

where the second equality follows from the total probability theorem.

For each $0 \leq k \leq N$, we have to consider all k-out-of-N user samples without replacement. We denote each one by $q_l^k = \left\{q_l^{k.1}, ..., q_l^{k.k}\right\}$ where $q_l^{k.i}$, $1 \leq i \leq k$ are user indices belonging to the current k-sample. The total number is $L(N,k) = \begin{pmatrix} N \\ k \end{pmatrix}$ and for $k=0$, $q_l^0 = \{\emptyset\}$. Furthermore, the complement of the index set q_l^k is the set containing indices of idle users. This set is denoted by $\hat{q}_l^k$ and its size is $N-k$. The probability of the users in q_l^k to transmit with all users in $\hat{q}_l^k$ being idle, equals $\alpha_{k,l} := \prod_{i=1}^{k} b_{q_l^{k.i}} \cdot \prod_{j=1}^{N-k}(1 - b_{\hat{q}_l^{k.j}})$. With this observation at hand, we have

$$CP^{(b)}(N,t) = \sum_{k=0}^{N} CP(k,t) \cdot \left(\sum_{l=1}^{L(N,k)} \prod_{i=1}^{k} b_{q_l^{k.i}} \prod_{j=1}^{N-k} (1 - b_{\hat{q}_l^{k.j}})\right) \tag{44}$$

which proves (10). Moreover, assuming $b_n = b$ for all users n in (44), yields the second CP expression in the proposition.

To prove (11), observe that all possible index set pairs $\left\{q_l^k, \hat{q}_l^k\right\}$, $\forall k, l$, form a sample space for an experiment of N users being present within a cell and having a probability each to transmit or not. Hence $\sum_{k,l} \alpha_{k,l} = 1$ and the following holds

$$\begin{aligned} CP^{(b)}(N,t) &\overset{(44)}{=} \sum_{k,l} CP(k,t) \cdot \alpha_{k,l} \\ &\overset{\textbf{Property 2}}{\leq} \sum_{k,l} CP(N,t) \cdot \alpha_{k,l} \\ &= CP(N,t) \end{aligned}$$

■

Proof: **[Proposition 2]** Let $\mathcal{F}^{(t)} := \{\mathbf{S}(0), \ldots, \mathbf{S}(t)\}$ be the information over the system realizations up to slot t. Obviously $\mathcal{F}^{(0)} \subseteq \mathcal{F}^{(t)}$ (formally we call $\left\{\mathcal{F}^{(t)},\ t \geq 0\right\}$ a filtration and $\mathcal{F}^{(0)}$ is a sub-σ-algebra of $\mathcal{F}^{(t)}$) and the tower property for expectations [17, p.88] holds. Hence,

$$\begin{aligned} \mathbb{E}\left[V(\mathbf{S}(t+1))\,|\mathbf{S}(0)\right] &\overset{Tower}{=} \mathbb{E}\left[\mathbb{E}\left[V(\mathbf{S}(t+1))\,|\mathcal{F}^{(t)}\right]|\mathcal{F}^{(0)}\right] \\ &\overset{Markov}{=} \mathbb{E}\left[\mathbb{E}\left[V(\mathbf{S}(t+1))\,|\mathbf{S}(t)\right]|\mathbf{S}(0)\right] \\ &\overset{(21)}{=} \mathbb{E}\left[D\left(V(\mathbf{S}(t)), \mathbf{A}(t)\right) + V(\mathbf{S}(t))\,|\mathbf{S}(0)\right] \\ &= \mathbb{E}\left[D\left(V(\mathbf{S}(t)), \mathbf{A}(t)\right)|\mathbf{S}(0)\right] + \mathbb{E}\left[V(\mathbf{S}(t))\,|\mathbf{S}(0)\right] \end{aligned}$$

and by repeating the process for $t, \ldots, 0$ and taking the limits for $t \to \infty$ at both sides we reach the result. ■

Proof: **[Proposition 3]** Consider the series in (22) up to a finite horizon $T+1$ and denote the related sum by $\mathcal{M}_T(V)$. We denote by $p_{s_{\tau-1}\to s_\tau}$ the transition probability from state $\mathbf{S}(\tau-1)$ to $\mathbf{S}(\tau)$. Then the expected drift term for some $\tau \leq T$ equals

$$\mathbb{E}\left[D\left(V\left(\mathbf{S}(\tau)\right), \mathbf{A}(\tau)\right) | \mathbf{S}(0)\right] = \sum_{\mathbf{S}(1)} \cdots \sum_{\mathbf{S}(\tau)} p_{s_o \to s_1} \cdot \ldots \cdot p_{s_{\tau-1}\to s_\tau} \cdot D\left(V\left(\mathbf{S}(\tau)\right), \mathbf{A}(\tau)\right) \quad (45)$$

where each summation is taken over the entire state space at time slot $1, \ldots, \tau$.

It can be observed that $p_{s_{\tau-1}\to s_\tau}$, which can be controlled by the actions $\mathbf{A}(\tau-1)$ taken at time $\tau-1$, appear in all summands of $\mathcal{M}_T(V)$, for $\tau \leq \hat{t} \leq T$ and not for $0 \leq t \leq \tau-1$.

Following this observation, problem (23) can be rewritten as

$$\begin{aligned} \textbf{min} \quad & \min\nolimits_{\mathbf{A}(T)\in\mathbb{A}} \mathcal{M}_T\left(V, \tilde{\mathbf{A}}\right) \\ \textbf{s.t.} \quad & \mathbf{A}(t) \in \mathbb{A},\ t \in [0, \ldots, T-1] \end{aligned} \quad (46)$$

where the optimal choice of actions $\mathbf{A}(T)$ in T define the transition probabilities $p_{s_T\to s_{T+1}}$. These do not appear in other terms and can be found by minimizing the expression for the drift at T. The optimal cost-to-go function equals

$$J\left(\mathbf{S}(T)\right) := \sum_{\mathbf{S}(T+1)} p^*_{s_T\to s_{T+1}} \left(V\left(\mathbf{S}(T+1)\right) - V\left(\mathbf{S}(T)\right)\right) \quad (47)$$

which may be easily verified to satisfy the recursion, $\forall \mathbf{S}(\tau-1) \in \mathcal{S}$:

$$J\left(\mathbf{S}(\tau-1)\right) = \min_{\mathbf{A}(\tau-1)\in\mathbb{A}} \sum_{\mathbf{S}(\tau)} p_{s_{\tau-1}\to s_\tau} \left(V\left(\mathbf{S}(\tau)\right) - V\left(\mathbf{S}(\tau-1)\right) + J\left(\mathbf{S}(\tau)\right)\right). \quad (48)$$

The expression hold as well, when we let the horizon $T \to \infty$. Thus taking $\tau \to \infty$ results in (24).

■

Proof: **[Proposition 4]** By Proposition 3, the optimal transition probabilities per time slot t are given by solving the problem

$$\min_{\mathbf{A}(t)\in\mathbb{A}} \sum_{\mathbf{S}(t+1)} p_{s_t\to s_{t+1}} \left(V\left(\mathbf{S}(t+1)\right) - V\left(\mathbf{S}(t)\right) + J\left(\mathbf{S}(t+1)\right)\right).$$

Now when minimizing the drift, only the transition $\mathbf{S}(t) \to \mathbf{S}(t+1)$ is considered by neglecting the term $J\left(\mathbf{S}(t+1)\right)$. This leads to a suboptimal solution, which is called “myopic“ in the sense that it takes just the next state transition into consideration and is ignorant of the cost over the horizon $t+1$.

■

References

[1] N. Abramson. The ALOHA system - Another alternative for Computer Communications. *Proc. AFIPS Fall Joint Comput. Conf.*, 27, 1970.

[2] A. Ephremides and B. Hajek. Information theory and communication networks: an unconsummated union. *IEEE Trans. on Inf. Theory*, 44, no. 6:2416–2434, Oct. 1998.

[3] G. Bianchi. Performance analysis of the IEEE 802.11 distributed coordination function. *IEEE JSAC*, 18, issue:3:535–547, Mar. 2000.

[4] P. Gupta, Y. Sankarasubramaniam, and A. Stolyar. Random-Access Scheduling with Service Differentiation in Wireless Networks. *INFOCOM 2005*, 3:1815 – 1825.

[5] G. Sharma, A. Ganesh, and P. Key. Performance analysis of contention based medium access control protocols. *INFOCOM 2006*, pages 1–12, Apr. 2006.

[6] 3GPP TS 36.300 (v 8.7.0) (May 2009) ”Technical specification group radio access network; Evolved universal terrestrial radio access (E-UTRA) and evolved universal terrestial radio access network (E-UTRAN); overall description” Release 8.

[7] M. Amirijoo, P. Frenger, F. Gunnarsson, J. Moe, and K. Zetterberg. On Self-Optimization of the Random Access Procedure in 3G Long Term Evolution. *Proc. IEEE Integrated Network Management-Workshops, 2009.*, pages 177–184, Jun. 2009, New York, NY, USA.

[8] B. Hajek and T. van Loon. Decentralized dynamic control of a multiaccess broadcast channel. *IEEE Trans. on Automatic Control*, AC-27, no. 3:559–569, June 1982.

[9] G. Dimic, N. D. Sidiropoulos, and R. Zhang. Medium Access Control - Physical Cross-Layer Design. *IEEE Signal Processing Magazine*, 4, Sep. 2004.

[10] L. Tong, Q.Zhao, and G. Mergen. Multipacket reception in random access wireless networks: From signal processing to optimal medium access control. *IEEE Comm. Magazine*, pages 108–112, Nov. 2001.

[11] Y. Al-Harthi and S. Borst abd P. Whiting. Distributed adaptive algorithms for optimal opportunistic medium access. *Mobile Netw Appl (Springer)*, 16, Issue 2:217–230, April 2011.

[12] 3GPP TS 36.213 (V 8.7.0) Technical specification group radio access network; Evolved universal terrestrial radio access (E-UTRA); Physical layer procedures, Release 8, May 2009.

[13] W. Feller. *An Introduction to Probability Theory and Its Applications, Volume I.* John Wiley & Sons, 1968.

[14] *Nonnegative matrices in the mathematical sciences*. SIAMClassics in Applied Mathematics, 1994.

[15] M. L. Puterman. *Markov Decision Processes: Discrete Stochastic Dynamic Programming*. Wiley & Sons, 2005.

[16] Dah-Ming Chiu and Raj Jain. Analysis of the increase and decrease algorithms for congestion avoidance in computer networks. *Computer Networks and ISDN Systems*, 17, North Holland:1–14, 1989.

[17] D. Williams. *Probability with Martingales*. Cambridge, 1991.

TABLES

TABLE I

GENERAL SELF-OPTIMIZATION ALGORITHM

STEP 1:	Gather empirical information $\mathcal{I}$ at the base station (measurements and user reports).
STEP 2:	Estimate certain unknown factors (see 1. - 3. above) of the objective function (27).
STEP 3:	Solve the resulting approximate optimization problem in (25).
STEP 4:	Broadcast action-related information $\mathcal{J}$ to the users.
STEP 5:	Calculate at the user side the required actions $A_n(t)$, $\forall n$, based on $\mathcal{J}$.

TABLE II

PARAMETER TABLE

Parameters	Value
Wireless Network	Single cell
User distribution	Uniform within cell
Number of users in cell	$\{1, 2, \ldots, 14\}$
Sequence pool size	10
Fixed Tx Power	250 mW
Power ramping step Δp	20 mW
Maximum Tx Power	500 mW
Path loss PL	$128.1 + 37.6 \log(D\ km)$ dB
Noise	-133.2 dB
SNR threshold	8 dB
Maximum effort M	5
Fixed backoff probability	$[0.5, 0.4, 0.3, 0.2, 0.1]$
Number of slots	15000 slots
Window length W	200 slots
Backoff factor A	$\{0.25, 0.5\}$
Access Function $f(m)$	m^{-1}
Power control factor δ_1	2×10^{-4}
Power control factor δ_2	8 mW
DMR^H	3.5%
DMR^L	2.5%

FIGURES

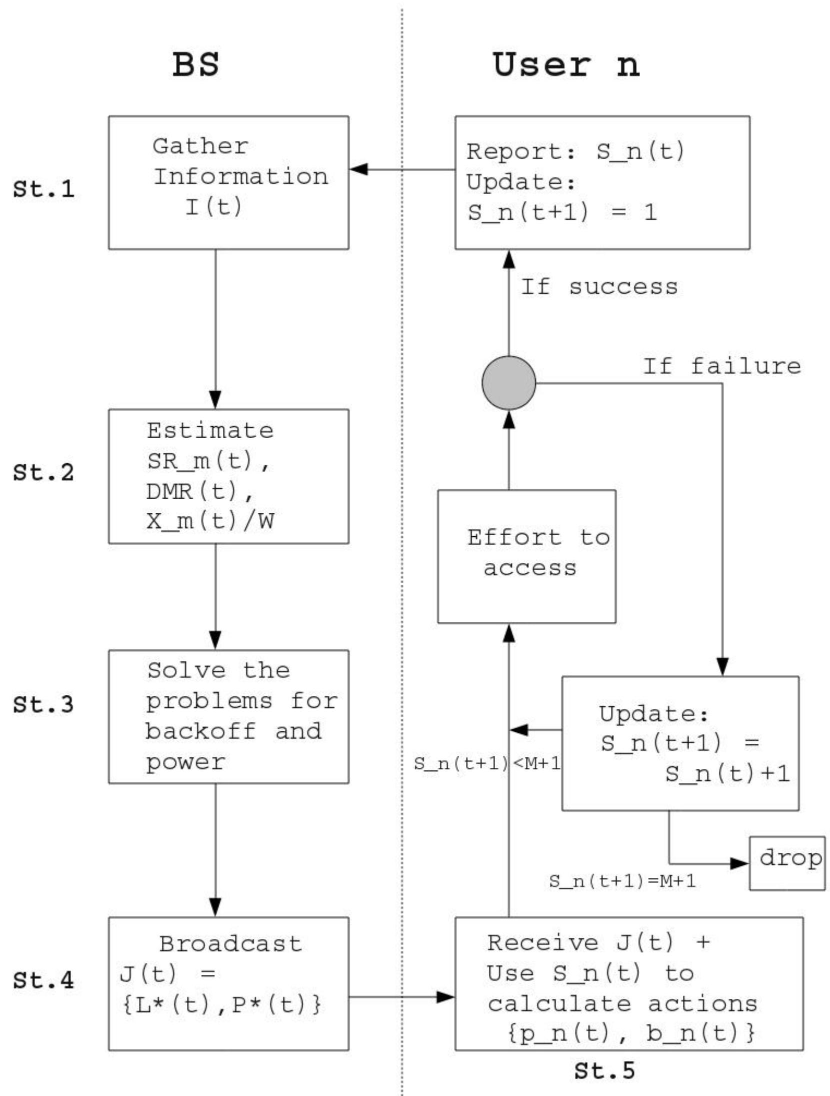

Fig. 1. Block Diagram of the suggested algorithm including the message passing between the BS and the mobile user

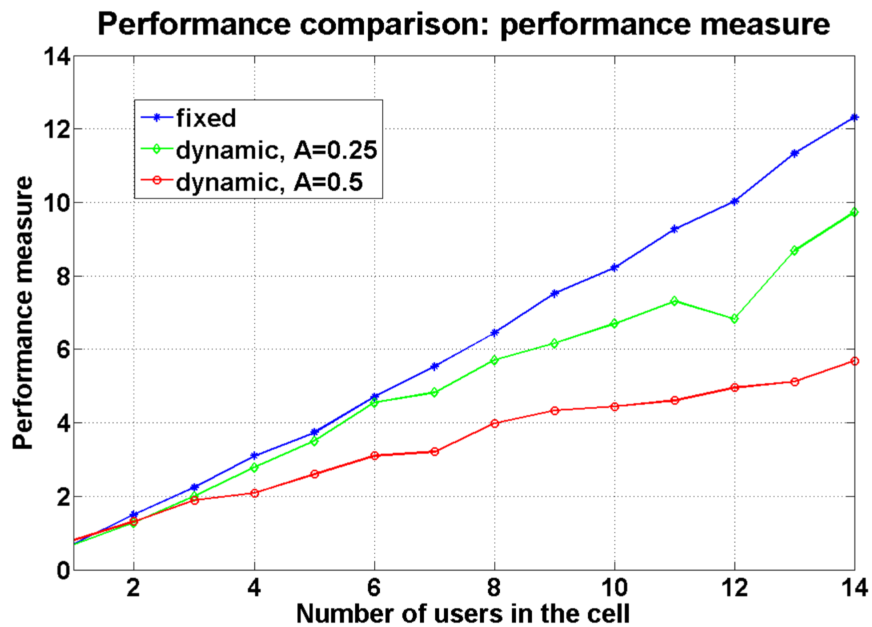

Fig. 2. Comparison of performance measure, equal to the chosen function V as $t \to \infty$. The measure improves with increasing idle probability bound $\mathcal{A}$.

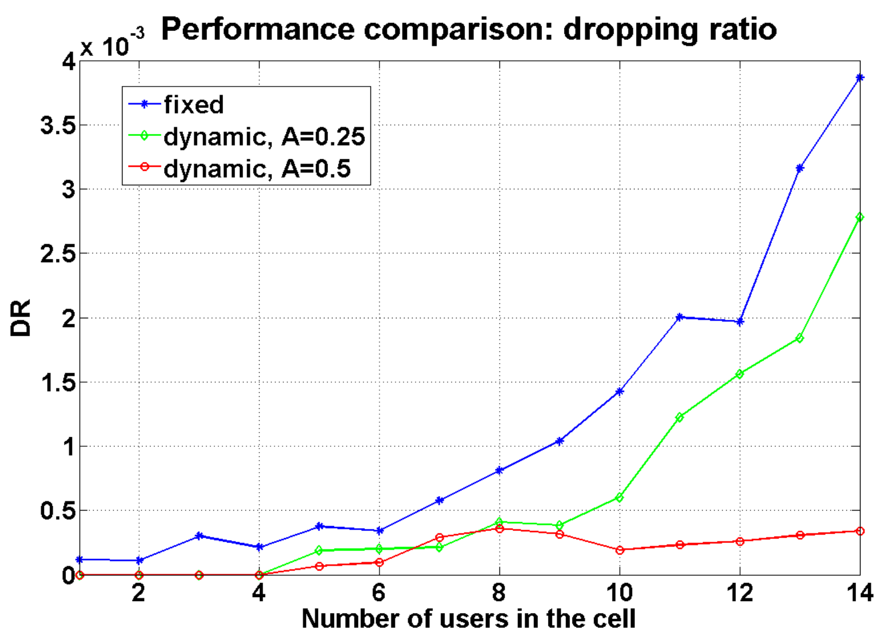

Fig. 3. Comparison of the average dropping rate (DR). The abrupt increase of the rate after a certain user number is an indicator that the system is not anymore stable for a further increase in the cell user number. Higher values of $\mathcal{A}$ can increase the point when the instability appears, at the cost of delay.

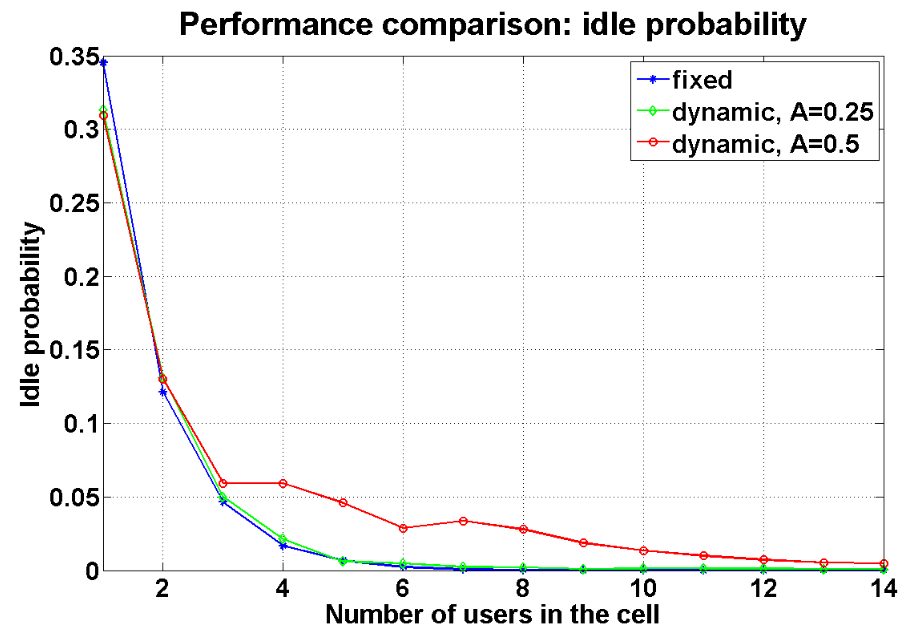

Fig. 4. Comparison of the average occurence of idle slot per scheme. The dynamic scenario with $\mathcal{A} = 0.25$ follows the chosen fixed one.

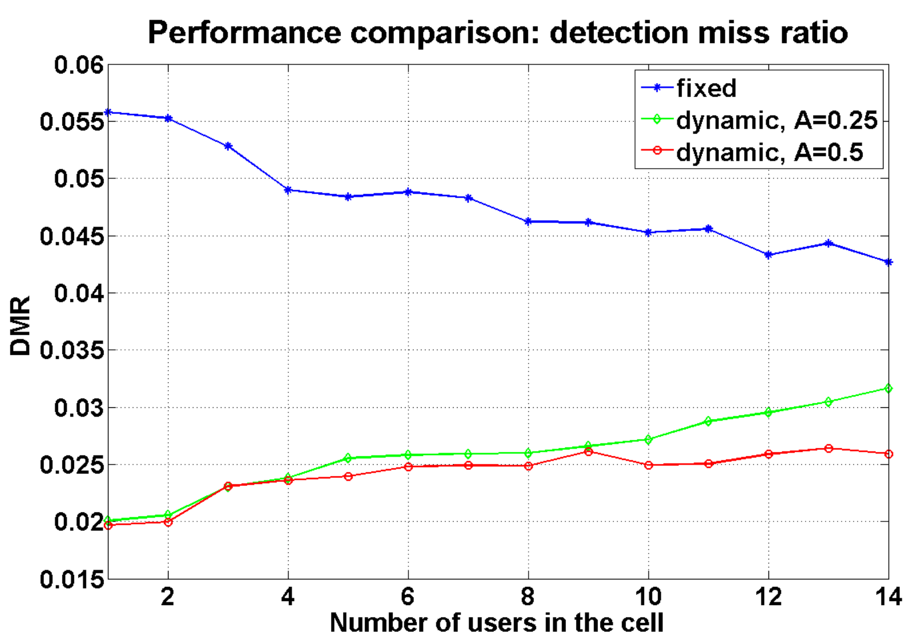

Fig. 5. Comparison of detection-miss rate (DMR) for the different schemes.

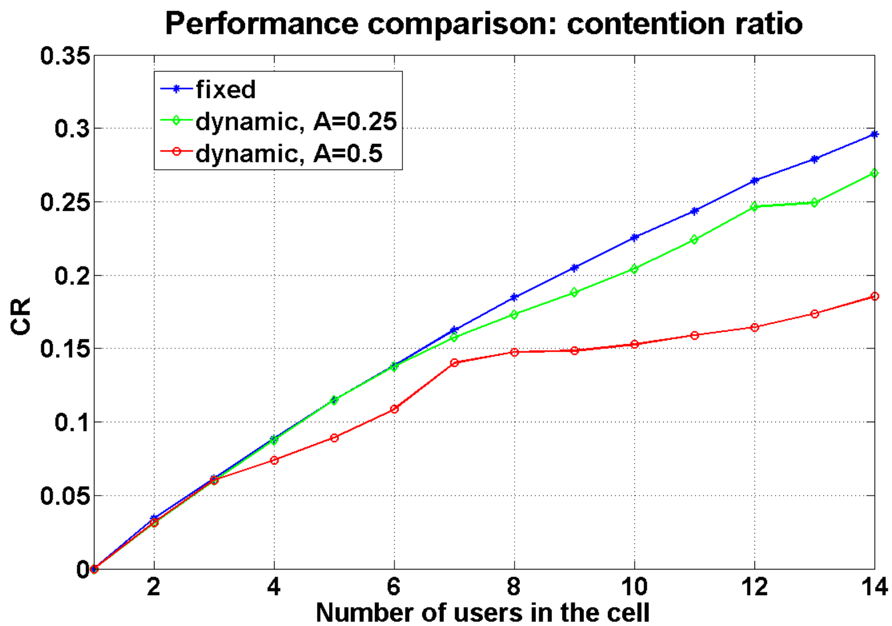

Fig. 6. Comparison of contention rate (CR) for the different schemes.

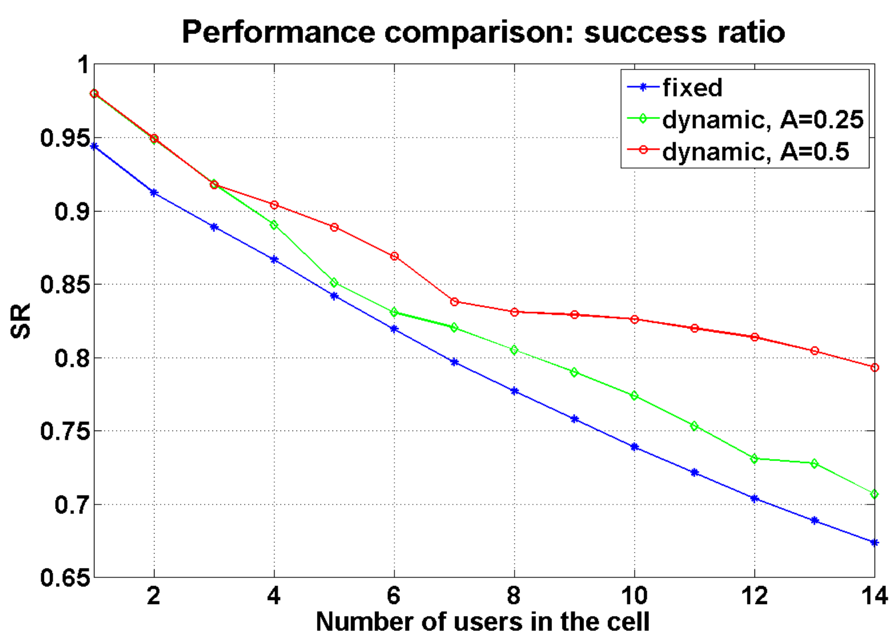

Fig. 7. Comparison of the overall success rate (SR) for the different schemes.

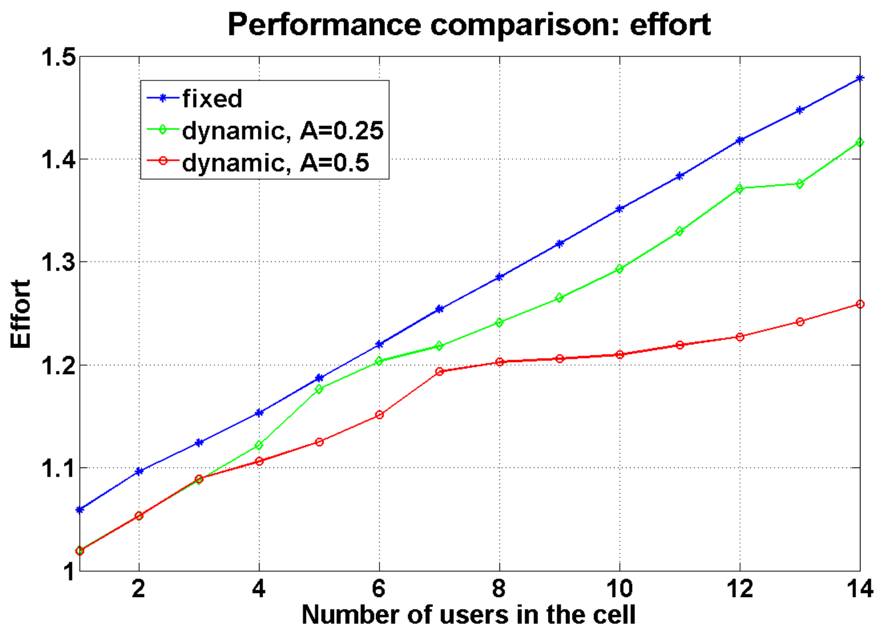

Fig. 8. Comparison of average number of efforts up to success (including the case of dropping).

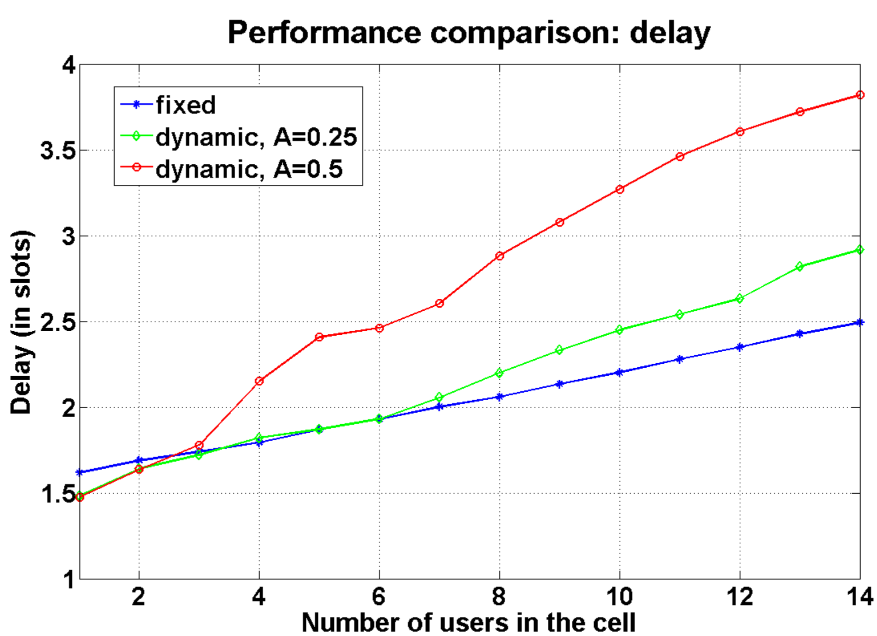

Fig. 9. Comparison of actual average delay up to success or dropping (alternatively: average time of user presence in the cell) in time slots.

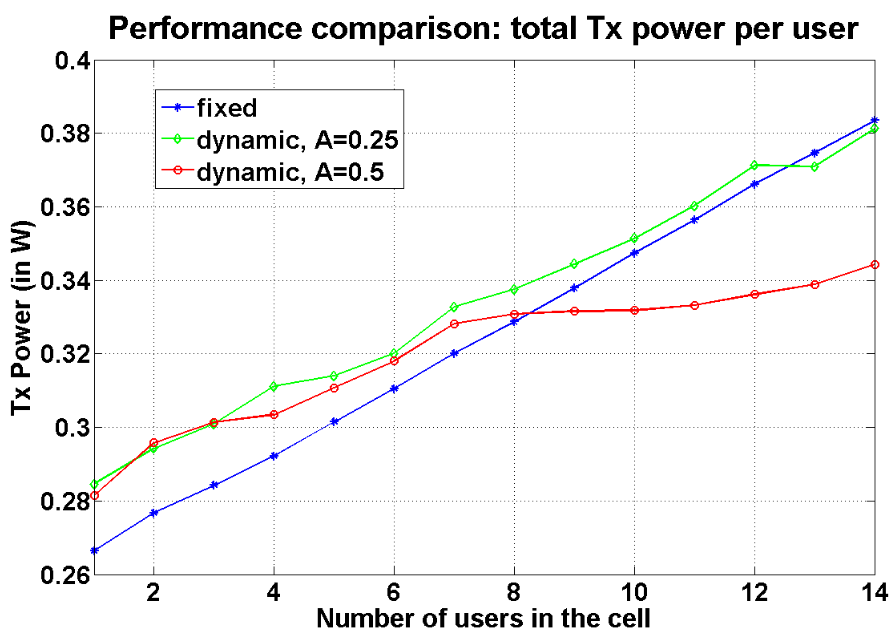

Fig. 10. Comparison of average total transmission power per user for the different schemes.